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An attempt to use the Stokes profiles I and V of the H_α line to determine the magnetic field configuration in three strongly magnetic CP stars

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1 Introduction

Polarization of radiation in the wings of the hydrogen Balmer lines in the spectra of hot chemically peculiar magnetic stars, caused by the Zeeman effect, was used earlier to detect the magnetic field of these stars and reveal its character. A series of papers (Borra and Landstreet 1973) were concerned with measuring the difference of intensity in the wings of the H_β line in right- and left-circularly polarized spectra in a 5 Å band, whose center was within 5 Å from the line center on both sides. It was then converted to the value of the shift of the Zeeman circularly polarized components to determine the effective (mean over the visible disk) longitudinal magnetic field B_e. The character of variation of this quantity vrs rotational phase permitted us to judge of the field geometry: a purely sinusoidal curve pointed to a dipolar configuration, a more complex phase curve was interpreted as the presence of higher harmonics.

The advantage of using hydrogen lines for the investigation of the magnetic field of CP stars consisted in that hydrogen is the basic component of the chemical composition of a star, there are no "hydrogen spots", whereas the results of measurements of the magnetic field from the profiles of other lines may be distorted by the surface chemical inhomogeneity.

The choice of the line $H\beta$ in the papers by Borra and Landstreet (1973) was chiefly due to the spectral sensitivity of light detectors accessible at that time.

The present-day methods of studying CP stars enable using high accuracy polarization observations in the red region of the spectrum with high spectral resolution and obtain the profiles of all four Stokes parameters within a spectral line (Wade et al. 2000).

Concurrently with the development of the techniques of observations (and even with some advance) a mathematical method of solution of an inverse problem was formulated, which makes possible to determine local (at each point of the surface) Stokes parameters and thereby define the configuration of both chemical inhomogeneities and magnetic field in a star (Goncharskij et al. 1982; Vasil'chenko et al. 1996). It was given the name of Doppler-Zeeman mapping (hereafter D–Z mapping).

In the present paper we discuss a possibility of using the profiles of the Stokes parameters of the Balmer line H_α for the D–Z mapping. Interest in this line is explained by the following reasons.

1. This is "the most longwave line", one of the strongest in the spectra of CP stars. The Lande factor is sufficiently large. As is known, the magnetic effects (Zeeman splitting) at each point of the star surface are proportional to the local vector modulus of the magnetic field and proportional to the wavelength squared, while the effect of rotation (Doppler effect) shifting the lines in the spectrum of each point on the surface of the rotating star is proportional to the first power of the wavelength. This makes advantageous to work in the region of the longest waves of the spectrum in D–Z mapping.

2. The region in the vicinities of the H_α line for hot B stars is poor in stellar lines of other elements and also in atmospheric lines. So, the H_α line itself is free from blending. At the same time, sufficiently far from the H_α line core there are several weak sharp atmospheric lines, which prove to be very usefull as references for the determination of the scale of wavelengths exactly coincident for spectra of different polarization, which is of prime importance for the procedure of D–Z mapping. Besides, in the far wing of the H_α line there located the unblended lines of the 2nd multiplet of ionized carbon 6578 Å and 6583 Å, which are strong in the spectra of some early B stars, and the unblended lines FeII 6524.73 which can be used for D–Z mapping.

Table 1:

		Star	
Data	HD 37776	HD 175362	HD215441
Spectrum	B2	B5	B6
T_e, log g	22000, 4.0	17000, 3.5	16000, 4.0
Period	1.53869	3.67375	9.488
vsini	80	38	
Telescope	6 m BTA SAO	3.6 m ESO	2.7 McDonald
	MSS, CCD	CCd	
Observer	I.I.Romanyuk	G.Mathys	C.John-Krull
B_s (max)	56 kG	(B_e=+5/-4 kG)	34 kG

2 Observations

We had at our disposal Zeeman spectra around the H_α line for 3 magnetic CP stars with strong magnetic fields. Two of them had been investigated earlier by the methods of D–Z mapping from metal lines (HD 37776 — Khokhlova et al. 2000; HD 215441 — Khokhlova et al. 1997), and HD 175362 by the method of moments.

The necessary data on the spectra used in the paper are collected in Table 1.

The last stage of primary reduction of the spectra — averaging of noises and normalization to the continuous spectrum for HD 37776 and HD 215441 and also preparation of the profiles of the Stokes I and V parameters for mapping were performed by one of the authors (VLK) by the program DECH-20 (Galazutdinov 1992) and the graphics program AXUM. Since for the echelle spectra the length of the spectral order does not completely overlap the interval covered by the H_α line wings and since the distribution of energy along spectral orders of the echelle multiplied by the spectral sensitivity of the CCD matrix is not a linear function of the wavelength within a single Balmer order, the drawing of a continuous spectrum for Balmer lines is a difficult task which is affected by a human factor. We have used the option of the program DECH-20 for visual interpolation of the shape of the continuous spectrum at the orders with narrow lines neighboring H_α. A recently made attempt by Tsymbal (2002) to develop a code for performing such an operation automatically and with higher accuracy should be noted. As it will be shown below, for our task the errors introduced by the manual operation prove to be insignificant.

Figs. 1, 2, 3 show the profiles I and V for the stars of Table 1. Apart from the profiles of the I parameter, the figures exhibit the intensity profiles Rr and Rl for the right- and left-polarized Zeeman spectra. The profile of the I parameter is computed as the half-sum I=1/2(Rr+Rl), while the profile V as their difference V=(Rr–Rl). Each spectrum of Rr and Rl was normalized to the continuous spectrum independently.

Co-locating Rr and Rl spectra by narrow atmospheric lines along wavelengths allowed accurate measurements of the shifts of the Zeeman components to be made and higher measurement accuracy of the I and V parameters to be achieved. For the star HD 215441 (Fig. 3) for which several Zeeman spectra of the same phase were available statistical accuracy of the I and V parameters (random error) has been calculated to be 0.3 %, which corresponds to the signal/noise ratio 300. The coicidence of the profiles to this accuracy makes it possible to conclude that the random error arising in plotting the continuous spectrum is negligibly small.

Consider the behavior of the observed H_α line. In the spectrum of each of the stars this line consists of three parts conditionally displayed in Fig. 4. They can be represented by trapeziums: 1 is the narrow and deep central part, 2 is the intermediate part and 3 are extended shallow wings. The length of the upper bases of these trapeziums depends on the temperature T_e of the star, i.e. on log g, and also on the magnetic field strength and *vsini*. The upper base of trapezium 1 for HD 215441 (16000, 4.5) is 2.5 Å, for HD 175362 (17000, 3.5) it is 3.2 Å, for HD 37776 (22000, 4.0) it is equal to 7.0 Å. For the listed in Table 1 max H_β values the principal part of the profile of the V parameters forms in the region of trapezium 1, i.e. corresponds to the region of the high steepness of the sides of the profile of the I parameter. In the region of trapezium 2 with the lower steepness of the sides one can see faint wings of the profile of the Stokes parameter V for the star HD 215441 and at some phases for the star HD 175362. In the region of trapezium 3 (with extended shallow wings of the intensity profile) the Zeeman shift of the profile Rr and Rl does not produce noticeable polarization. Thus, the most informative part of polarization profiles for mapping by the Stokes parameters V, Q and U is the spectrum region located inside trapezium 1.

Now assume that when normalizing (when locating the continuous spectrum), a systematic error arose equal for all Rr and Rl profiles. In a small portion of the spectrum, equal to the upper base of trapezium 1, the steepness and curvature of the continuum can be neglected. The error of the value of the normalization factor will have but little effect on the profile of the V parameter since this parameter is a differential quantity.

As one can see from Figs. 1–3, the profile of the V parameter for all three stars is extremely "expressive" and prompted burning desire to use them for D–Z mapping the magnetic field by the method, developed in (Vasil'chenko et al. 1996, hereafter VSKh). Let us note some features of the line H_α profiles for each star listed in Table 1.

HD 215441. In papers on the study of the magnetic field of this star (Landstreet et al. 1989, Khokhlova et al. 1997) the division of narrow metal lines into π and σ components was used. Obvious predominance of a dipole component in the magnetic field configuration was also shown. Unfortunately, we had several successive averaged Zeeman spectra of H_α only for 4 phases, one of which was rejected because of an evident defect.

As is seen in Fig.1, the division of Zeeman circularly polarized σ-components leads to splitting of the I profile and gives the impression of an emission peak in the line center (Fig. 1a). Without polarization observations such a star could be attributed to B_e stars with weak H_α emission.

HD 175362. The H_α line shows a strong distinct crossover effect at phases separated by an interval $< 180°$, which attests a more complicate character of the strong large-scale magnetic field as was noted earlier in the paper by Mathys (1995).

HD 37776. This fast-rotating star with the strongest surface magnetic field from all known CP stars and a strong quadrupole component (Khokhlova et al. 2000) shows a relatively weak V profile of H_α. According to observations by Leroy (1995) and Romanyuk et al. (1998), the practical absence of linear polarization in the integral spectrum of this star could be explained by a complicated configuration of the magnetic field.

3 A possibility of using the hydrogen line H_α for D–Z mapping

To determine local profiles of the Stokes parameters, the VSKh method uses analytic solutions of the transfer equation — the relation between local profiles of the Stokes parameters and the local vector of the magnetic field. In the previous papers the profile of I parameter in the absence of magnetic field is given by an analytic solution of the transfer equation for the Miln-Eddington model with a depth-constant ratio of a selective coefficient to a continuous one. In our program (VSKh) this profile is replaced by a more realistic profile obtained by a numerical solution of the transfer equation on the basis of a suitable fitted model atmosphere. Such a replacement is fulfilled with the help of approximation formulae with specially chosen parameters (see 3.2).

3.1 Model atmosphere

When using the code (VSKh) for the D–Z mapping of the magnetic field by the line H_α we have to solve the problem of analytic representation of local profiles of the Stokes parameters of this line arising in the magnetic field. To use the analytic representation of the Stokes parameters profiles, we must calculate the I parameter profile in the absence of magnetic field. Keeping in mind that we will use the profile in the trapezium 1 region (Fig.1), where the informative part of the V parameter is formed, we must calculate first by a numerical solution of the transfer equation the profile central part, which is formed in the uppermost layers of the star atmosphere, and then represent it in an analytic form again.

For the numerical solution of the transfer equation a knowledge of the star model atmosphere is required. Up to date this is a weak point in any method of the D–Z mapping. There is yet no physically justified way to calculate a model atmosphere of a star with a magnetic field and ohmic anomalies so that would be possible to include this link into an inverse problem and to find a local physical model atmosphere by the method of iterations in the course of its solution. That is why usually, suitable "standard" LTE model atmospheres are accepted in the D–Z mapping method. For computing the uppermost layers of the model we used a version of the Kuruz ATLAS program modified by Tsymbal (2002). Its distinction is that the upper layers (up to $\log \tau \sim 10^{-10}$) are calculated more carefully and that the model convergence is controlled till attaining two criteria simultaneously: a condition of a constant integral flux and a condition of radiative equilibrium. The program SYNTH$-\mu$ by Tsymbal (2002) computing a synthetic spectrum allows output of an average depth

Table 2:

Transition	Zeeman structure	K
2P1/2-2D3/2	7 < 1000 > 75(250)87(750)	0.3335
2P3/2-2D3/2	27 < 1000 > 53(300)107(400)160(300)	0.0666
2P3/2-2D5/2	7 < 600 > 20 < 400 > 100(50)113(300)127(150)140(50)	0.6000

K is a relative oscillator strength for each transition

of formation (τ Ross) for each point of the profile. Table 2 presents an example of a computer job for model atmosphere of our stars.

Synthetic spectra in the vicinity of the lines H_α and H_β for a set of models were computed with this program. By comparing them to observational profiles the model atmosphere parameters for HD 215441 were estimated, which is shown in Table 1. The comparison of the profiles is presented in Fig.5 and 6. The fit of synthetic profiles with observational ones is rather good in the line wings, but it is naturally bad in the inner part of the profile, where the strong influence of the magnetic field on the line profile (and, possibly, on the structure of the upper atmospheric layers, too) remained unaccounted. Note, however, that the observed profiles of the Stokes I parameter practically do not depend on the phase, while the profiles of the V parameter appreciably change the amplitude with remoteness from the magnetic pole.

3.2 Selection of parameters for an analytic representation of local profiles of the Stokes parameters

In the analytic expression of the local profiles of the Stokes parameters in the D–Z mapping method (Vasil'chenko et al. 2000, hereafter VSKhII) the absorption factor is computed of the each Zeeman π and σ component with consideration for multiple structure of the line H_α. In the absence of field all components of the multiplet H_α coincide in wavelength, but the Zeeman structure of each component of the multiplet is different (Table 3).

The program "find RC" coded by D.Vasil'chenko selects parameters of the analytic representation of the local profiles in formulae [15,16], which represent the local profile of the I parameter in the absence of magnetic field for μ at different intensity well enough. For a chemically nonuniform surface the intensity difference is related to the difference of local element abundances. In the case of the line H_α in the presence of magnetic field each component of the Zeeman structure must be well represented analytically.

Rc= $C_1(1 - e^{C_2\tau})$, where C_1 is the central depth of a very saturated line, and C_2 is the properly chosen numerical parameter (Khokhlova et al. 1996, or VCKhII, formula 16). Later it was found that the best approximation for Rc is the formula $\mathrm{Rc}(\tau) = C_1\ (1 - 1/(1 + C_3\tau)^{-C_2})$, where τ is proportional to the number of absorbing atoms that determine the line intensity. This formula is more precise. It approximates the local profiles in a wide range of τ. The program "find RC" coded by D.V.Vasil'chenko automatically finds the approximation parameters C_1, C_2 and C_3. An example of selection of the parameters is shown in Fig. 7.

3.3 Allowing for the relationship between local profile and cosθ

In the D–Z mapping method (VSKh) the dependence of local profiles on an instant value of the angle θ is allowed for by multiplication of the local profile at $\theta = 0$ by a function U2(θ) in the integration element. I.e. it is supposed that the form of local profile does not depend on cos θ. In fact, as the results of numerical solutions show, the transfer equation for specific intensities is not satisfied (this is supposed for all H_α profile). However, as shown below, such an assumption can be acceptable for a central part of the profile (±1.5 Å from center) of interest to us.

To find the function U2(θ), the following procedure is applied: by a numerical solution of the transfer equation for a corresponding model atmosphere an array of profiles R(λ, μ) is computed, where μ cos(θ)R(λ, μ) is a computed profile (depth), which is then normalized by means of dividing by $R(\mu = 1, \lambda)$. To cover all possible intensities of the Zeeman components of the multiplet, such arrays are computed for a set of factor K values from 1.5 to 0.002.

Looking through the columns of this file, one can see that in the λ range of 1.5 Å from the line center the normalized values are only slightly λ-dependent, and an average value can be fixed in this range for each μ.

The relation between these average values and μ is the desired function U2(θ) in this range. Clearly, at a big distance from the line center (in wings) this function is inapplicable. The computations of H_α profiles for our model shown that the function U2(θ) found in such a way is cubically dependent on μ.

Fig. 8 presents a comparison of selected linear, square and cubic functions U2(θ). Fig. 6 shows a comparison of profiles computed by a numerical solution of the transfer equations and by analytic approximation formulae with the use of parameters found above.

4 The D–Z mapping of the star HD 215441 by the line H_α

Unfortunately, we have the perfect Zeeman spectra in the H_α range only for 4 phases. As a consequence, and because of a big inclination of the axis of rotation ($i = 30°$) an observer can see only a surface part shaded in the figure. The observed profiles of the Stokes I and V parameters are integrals of local profiles over this shaded surface.

Since we are looking for a solution in the form of the sum of a dipole, a quadrupole and an octopole with arbitrarily oriented axes, in process of iterations the program arranges them over the whole star surface in such a way that the coincidence of computed and observed profiles would be provided. In so doing, the positions of the positive and negative field maxima and the maps of the normal and tangential field vectors are computed. The high profile precision in the spectrum of HD 215441 makes possible to achieve a small discrepancy and to study the question of precision and unambiguity of the obtained model.

Since our program of the D–Z mapping keeps free the orientation of the dipole axes, then as the order increases, a possibility to get into secondary minima during iterations also enhances. That is why we have to vary the task parameters (in particular, the initial approximation) and to compare results obtained independently from different observational data. When speaking about the unambiguity of obtained solutions, we should compare the obtained maps, but not the parameters of the obtained multipoles.

Fig. 8 a, b, c presents the maps of normal and tangential field components obtained by three phases of the line H_α Zeeman spectra (in Fig. 9). An agreement between observed and computed data is obvious.

From the editors

This is an abrupt end of the report found by relatives of Vera L'vovna Khokhlova who died before her time. Vera L'vovna was bringing this report for reading at our Conference, but on the way to Zelenchukskaya she had serious problems with the health that she was not able to overcome.

In spite of the fact that we found only a part of the report, the editors decided to print it with a minimum number of corrections (only evident misprints were corrected). We found it possible to make captions for the figures not mentioned in the text and to find corresponding references, not all of which are designated in the manuscript.

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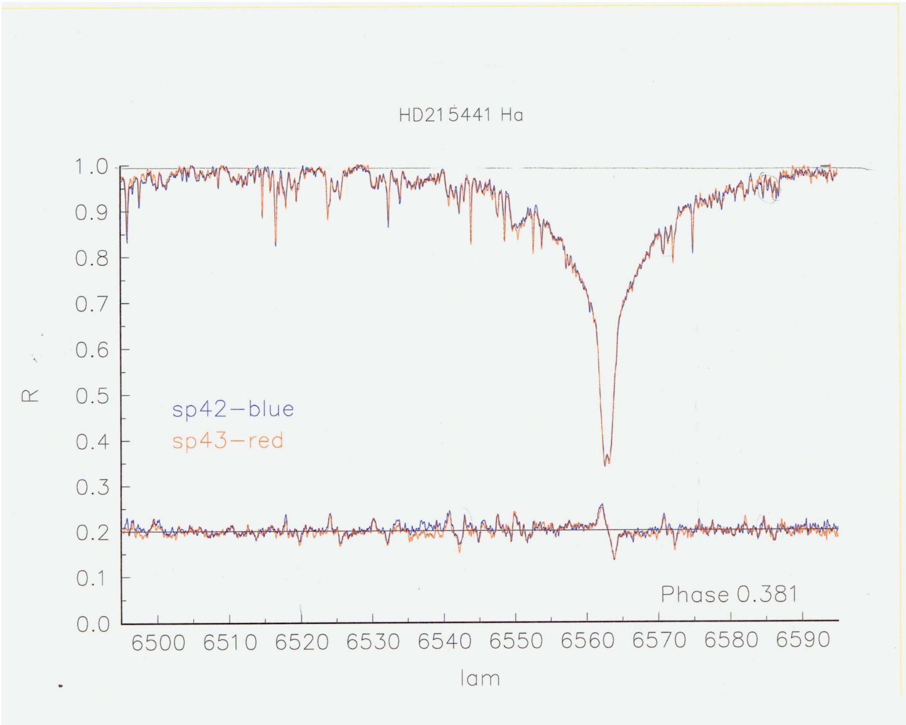

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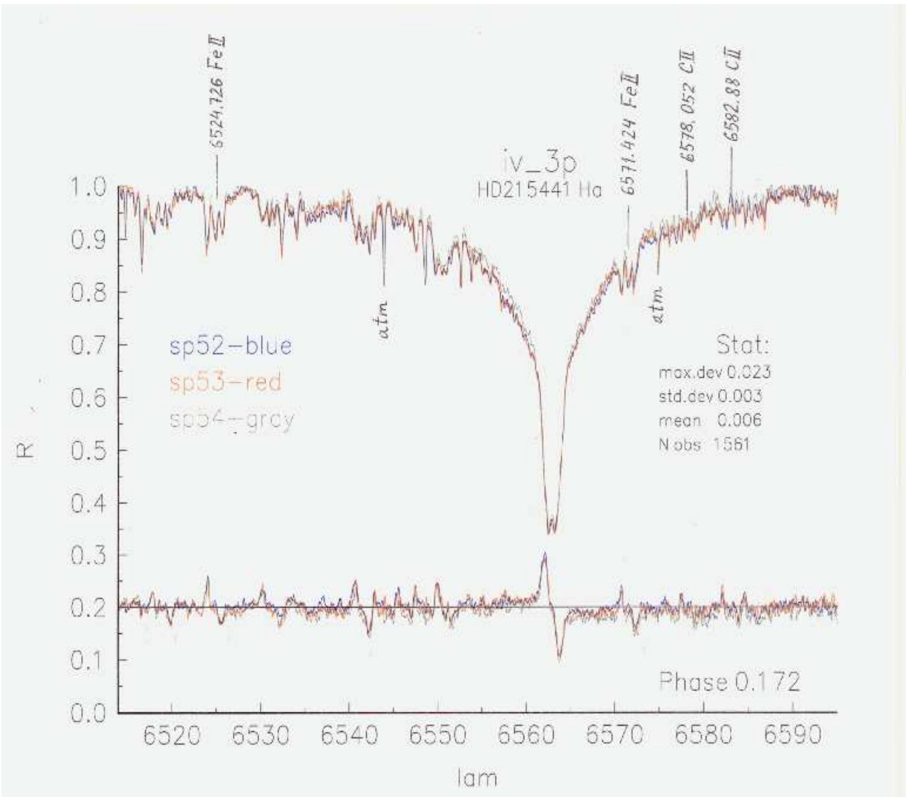

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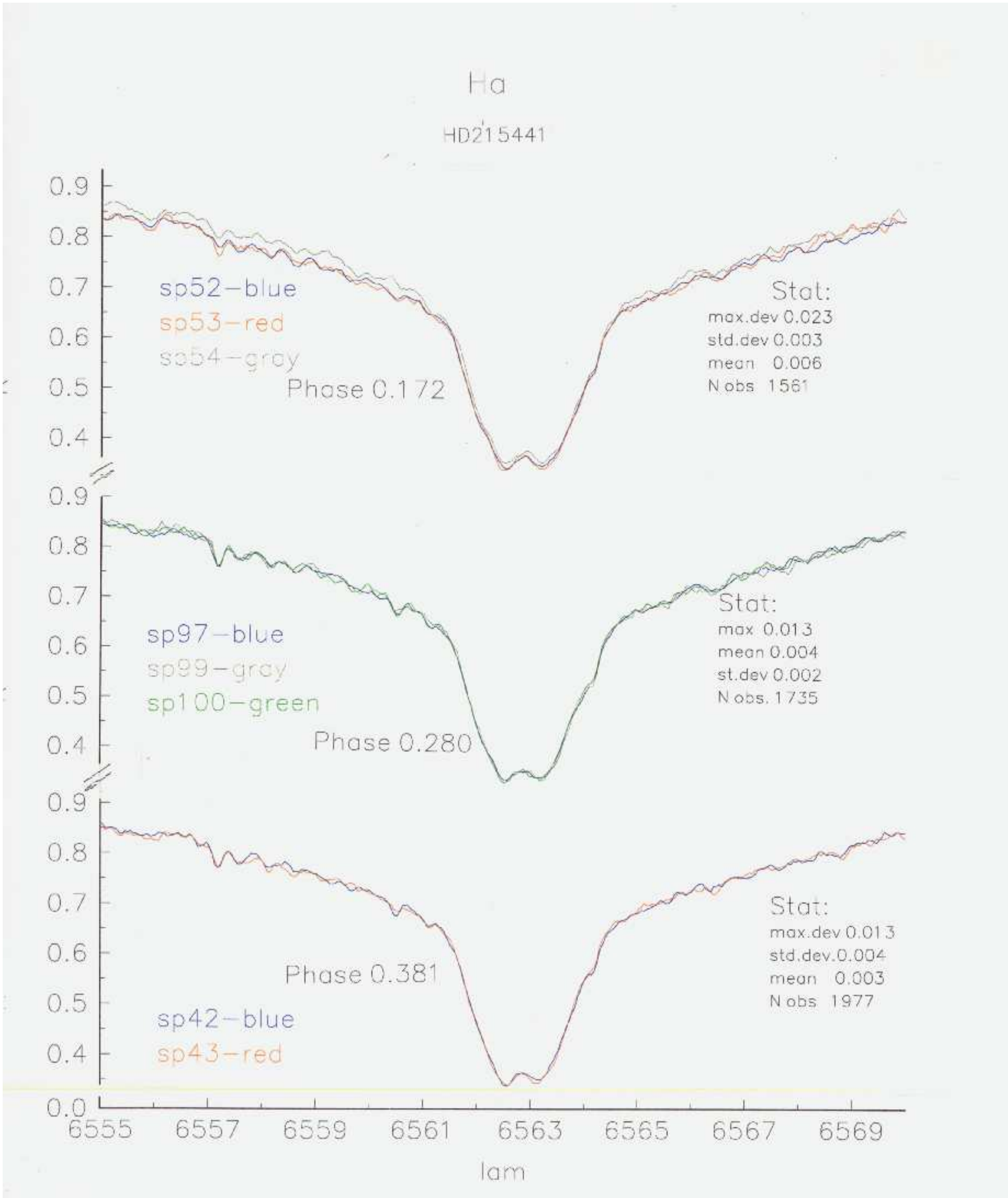
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sp53—red
sp54—gray
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std.dev 0.003
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N obs 1561
sp97—blue
sp99—gray
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Phase 0.280
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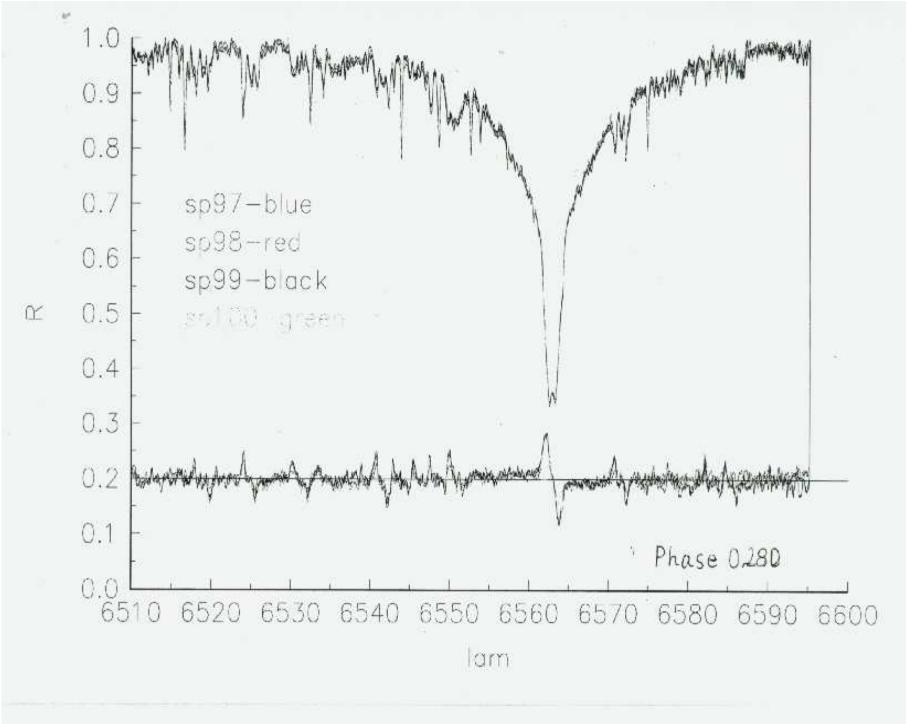

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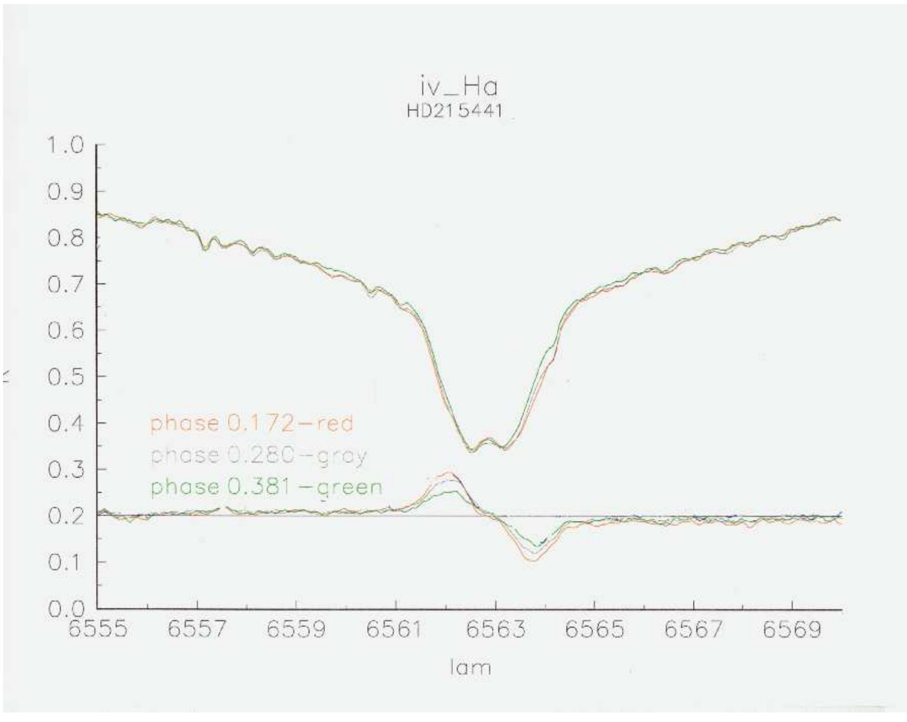

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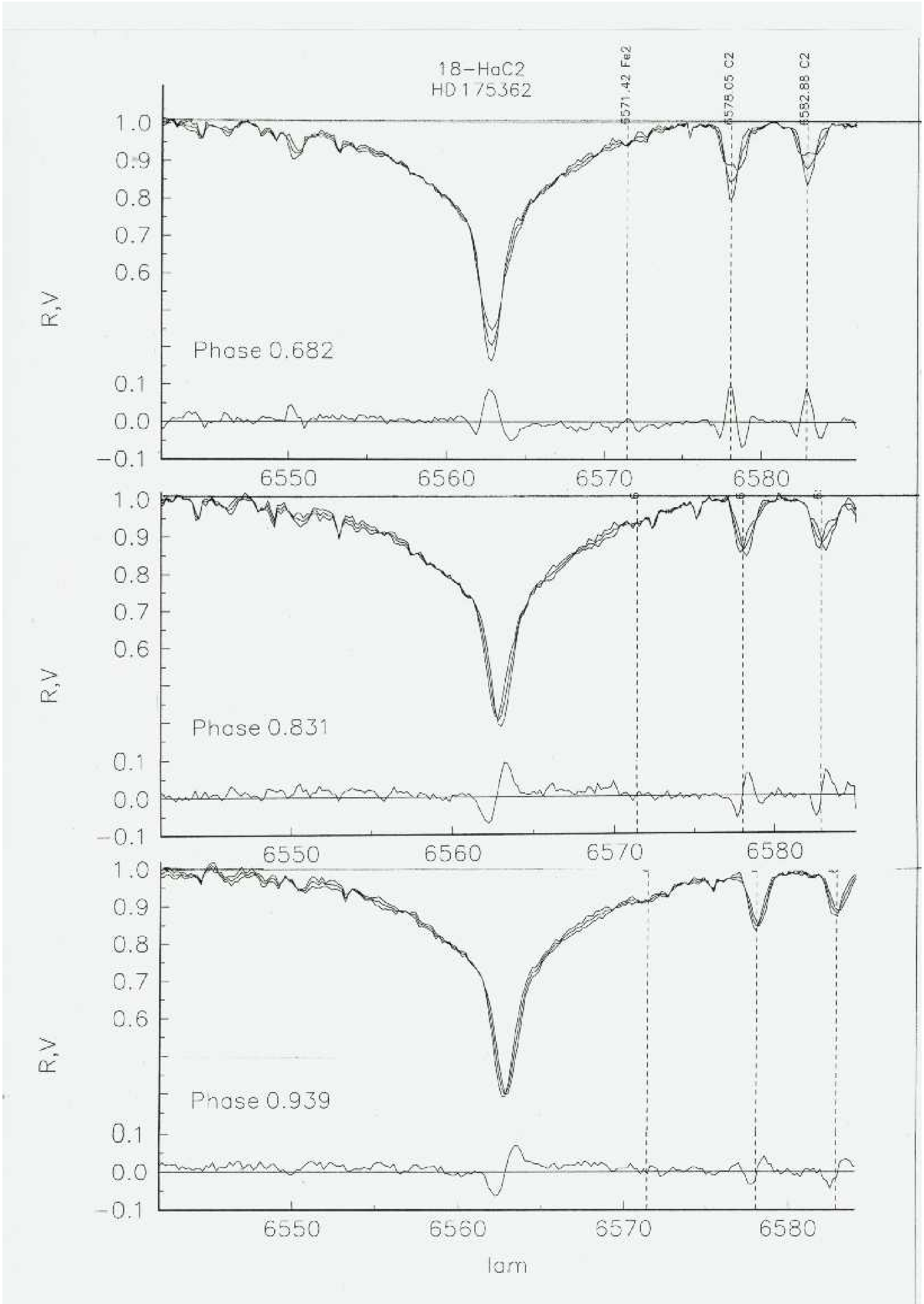
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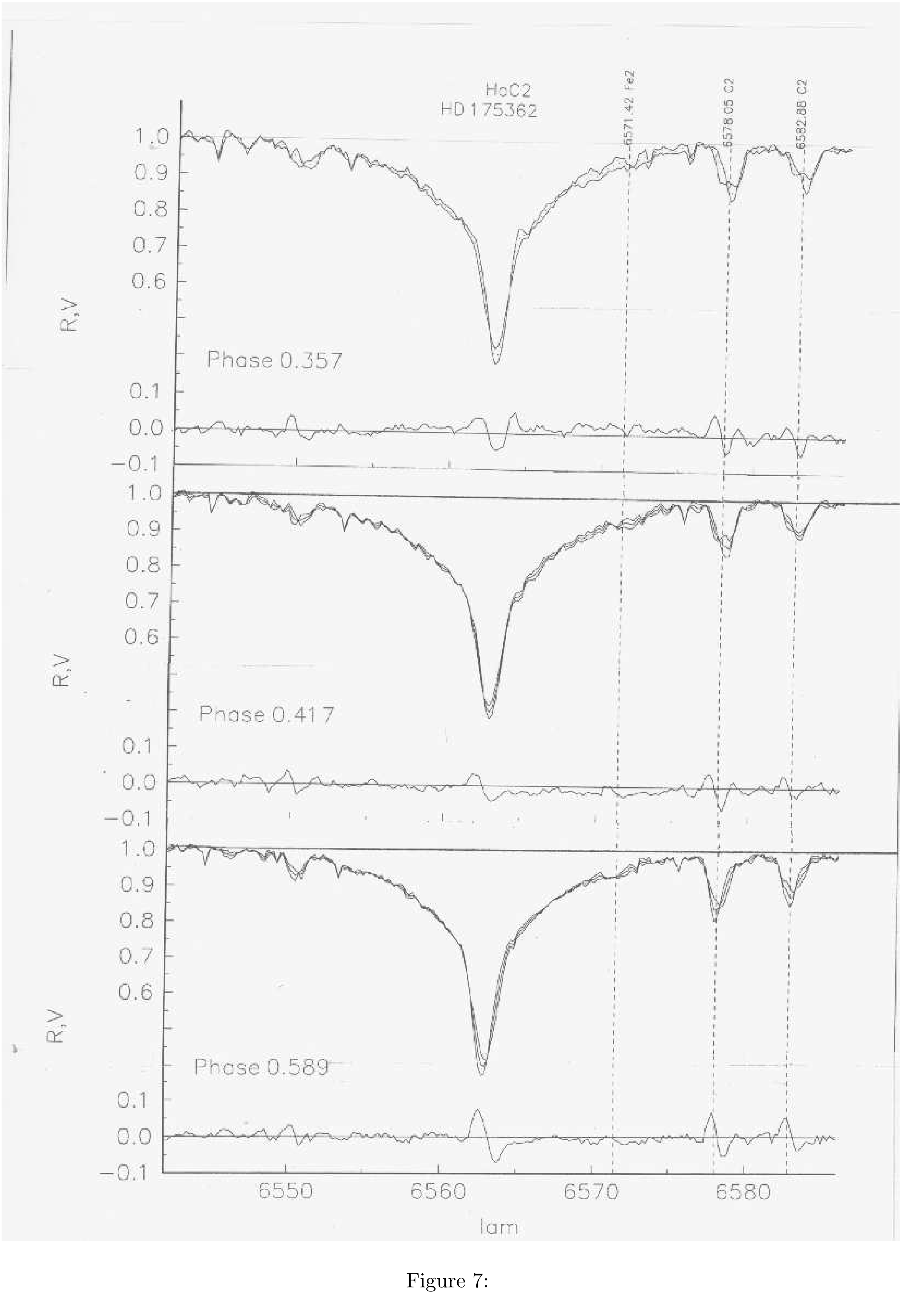
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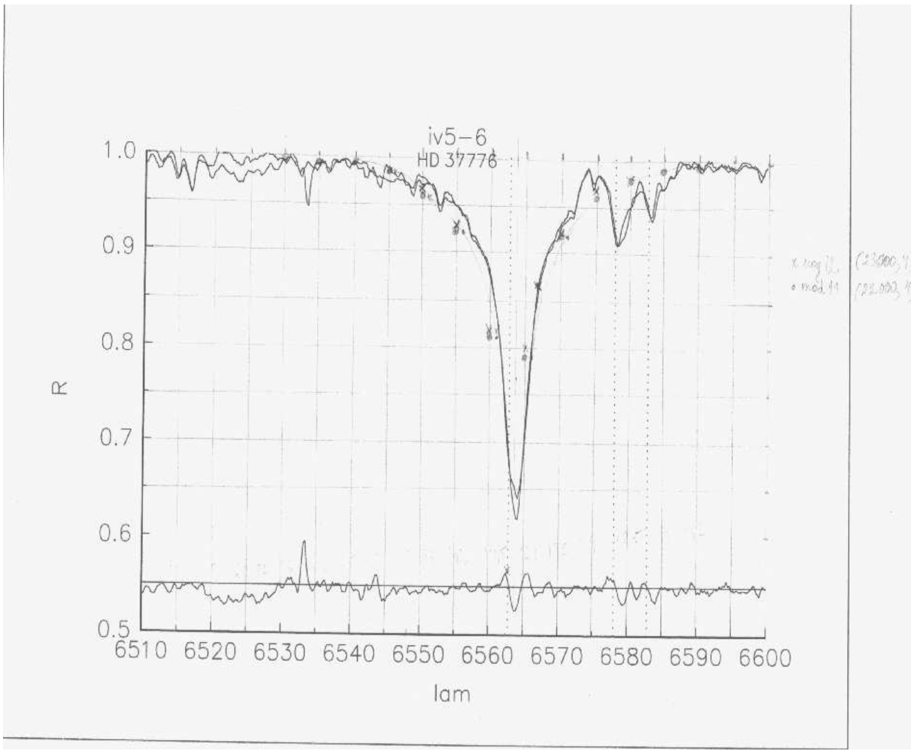
Figure 7:



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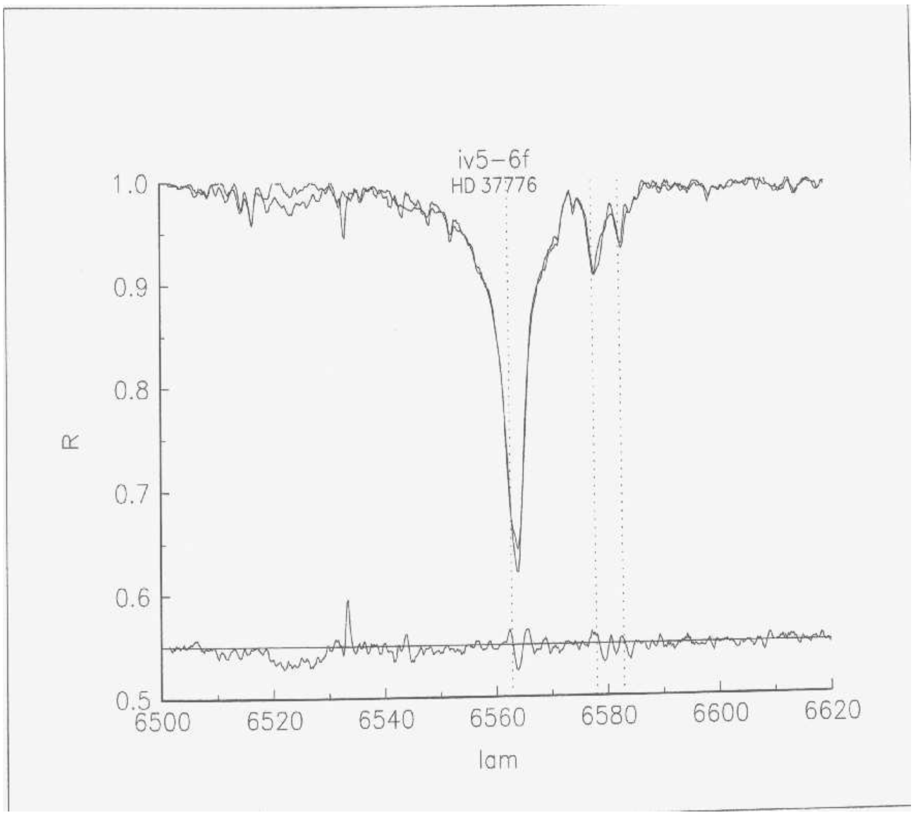

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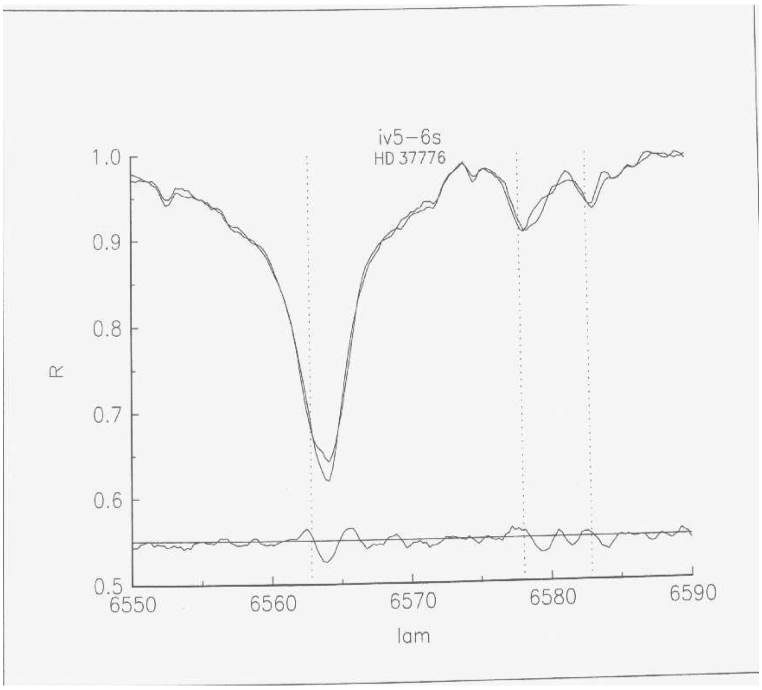

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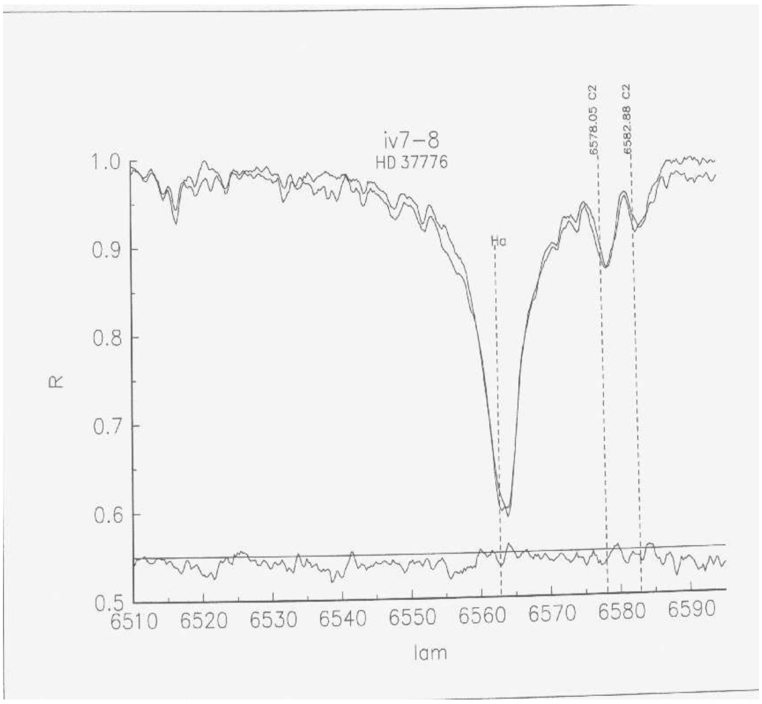

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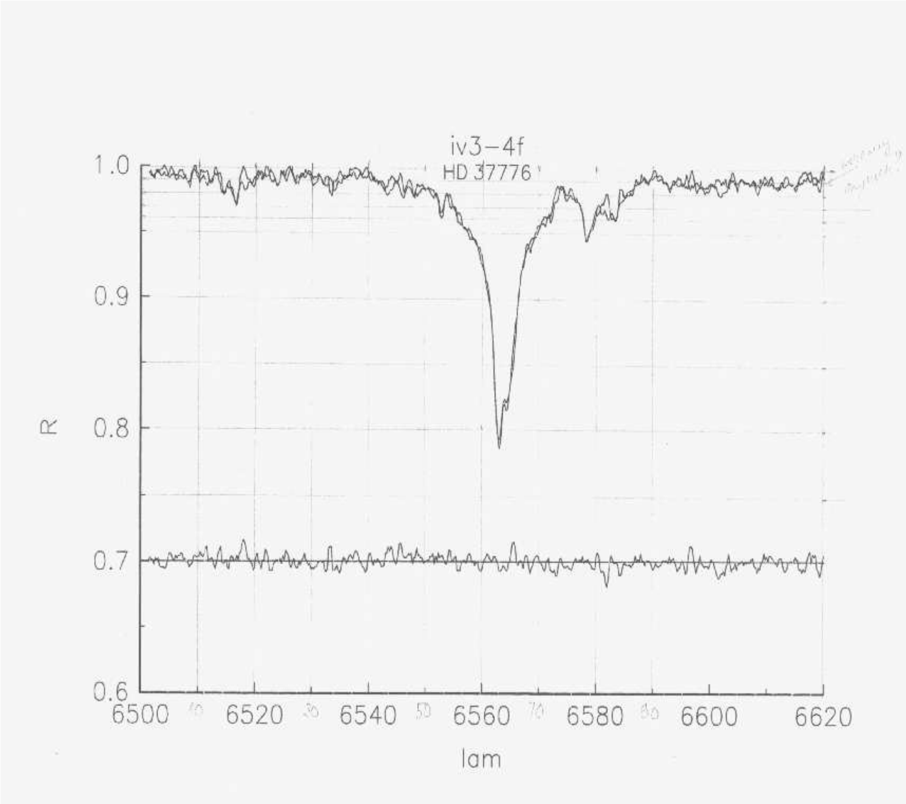

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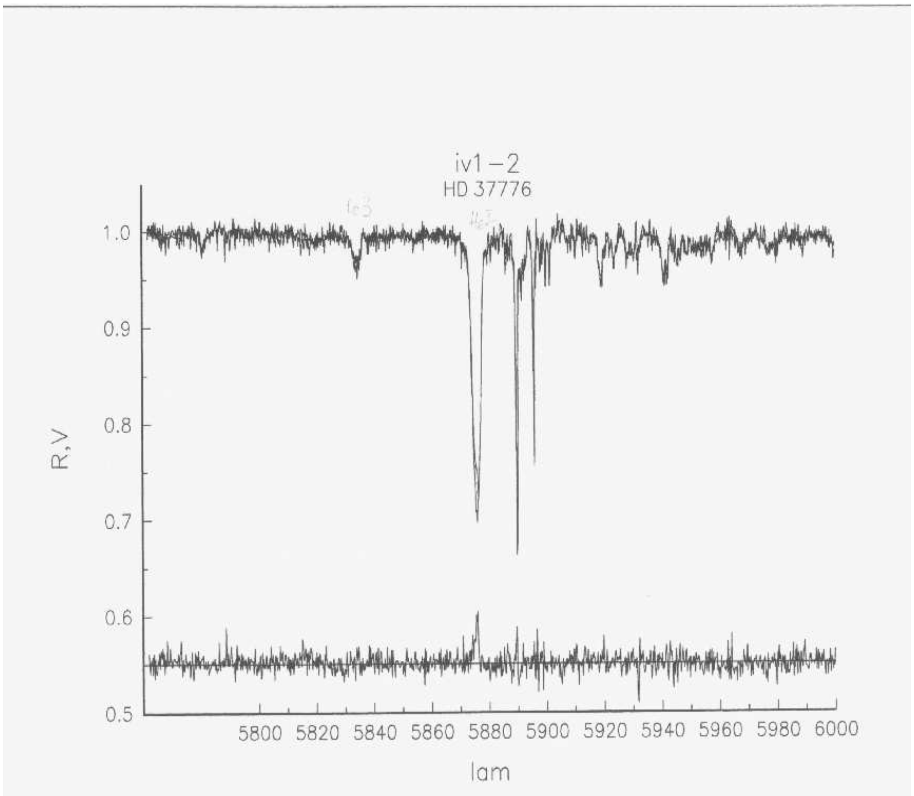

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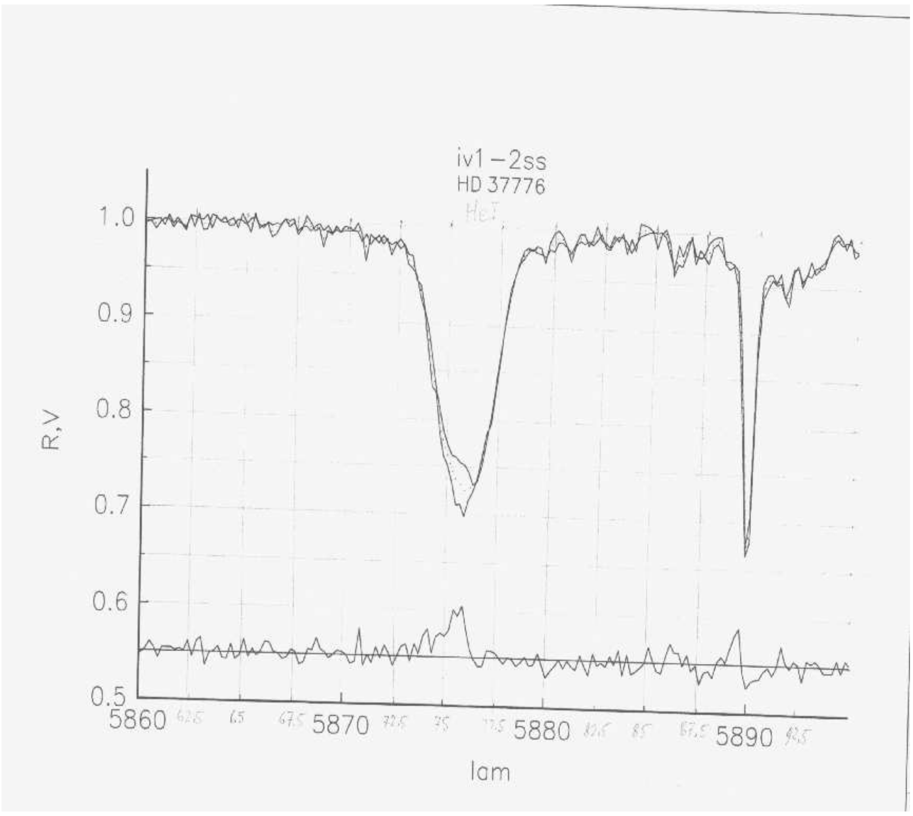

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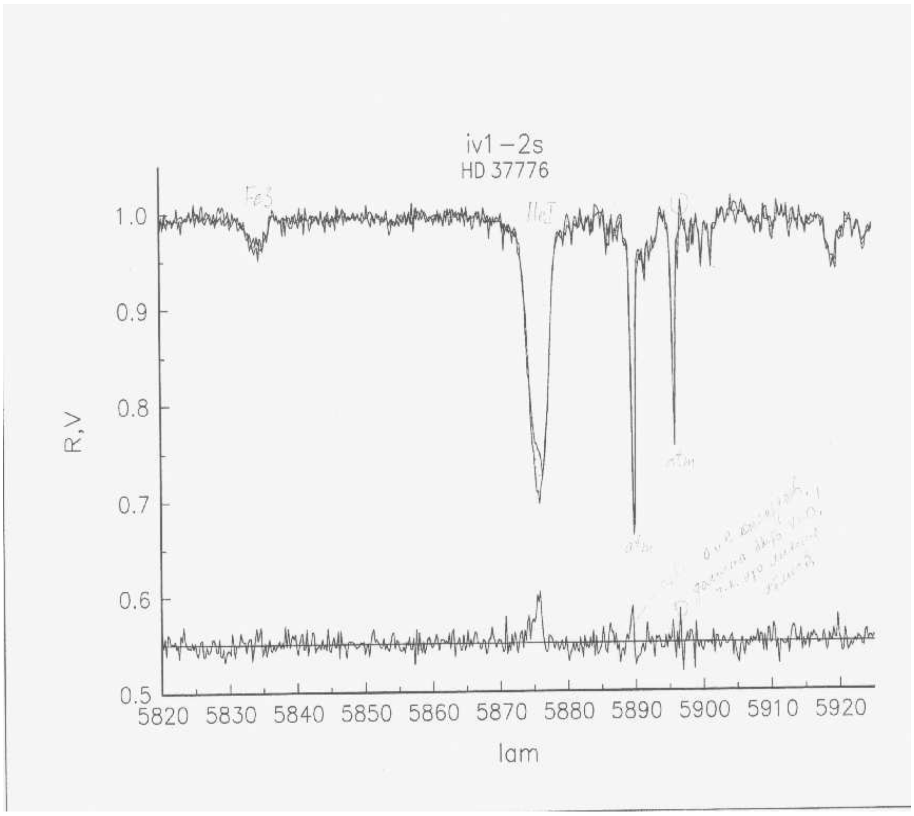

Figure 15: